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Book Reviews

Iterationsverfahren, Numerische Mathematik, Approximationstheorie.
Edited by L. Collatz, G. Meinardus, H. Unger and H. Werner.
International Series of Numerical Mathematics, Vol. 15. Birkhäuser Verlag, Basel, 1970.
257 pages, price: sFr. 36.—.

This volume contains lectures presented at three Oberwolfach meetings.

1. Meeting on non-linear problems of numerical mathematics, 17–23 November 1968.

Brosowski, B., K.-H. Hoffmann, E. Schäfer und H. Weber: Stetigkeitssätze für metrische Projektionen.
Brosowski, B., K.-H. Hoffmann, E. Schäfer und H. Weber: Metrische Projektionen auf lineare Teilräume von C_0 (Q, H).
Frehse, J: Über die Konvergenz von Differenzen- und anderen Näherungsverfahren bei nichtlinearen Variationsproblemen.
Laasonen, P: Über einige Lösungsverfahren nichtlinearer Gleichungssysteme.
Lancaster, P: Spektraleigenschaften von Operatorfunktionen.
Mayer, H.: Abschätzungen für den Defektvektor der Lösung eines linearen Gleichungssystems bei Ungenauigkeiten in den Ausgangsdaten und numerische Auswertung dieser Abschätzungen.
Nitsche, J.: Konvergenz des Ritz-Galerkinschen Verfahrens bei nicht-linearen Operatorgleichungen.
Nixdorff, K.: Nichtlineare Rechenmethoden der Peiltechnik.
Nixdorff, K.: Bemerkungen zur Anwendung der harmonischen Balance.
Reimer, M: Semidefinite Peano-kerne stabiler Differenzenformen.
Wetterling, W.: Über Minimalbedingungen und Newton-Iteration bei nichtlinearen Optimierungsaufgaben.
Zeller, K.: Newton-Čebyšev-Approximation.

2. Meeting on numerical methods in approximation theory, 8–14 June 1969.

Anselone, P. M.: Abstract Riemann Integrals, Monotone Approximations and Generalizations of Korovkin's Theorem.
Cheney, E. W. and K. Price: Minimal Interpolating Projections.
Collatz, L.: Approximationstheorie und Anwendungen.
Gilbert, R. P.: Integral Operator Methods for Approximating Solutions of Dirichlet Problems.
Haussmann, W.: Mehrdimensionale Hermite-Interpolation.
Locher, F., und K. Zeller.: Approximation auf Gitterpunkten.
Lupas, A.: On the Approximation by Linear Positive Operators.
Smith, L. B.: Using Interactive Graphical Computer Systems on Approximation Problems.

3. Meeting on iterative methods in numerical mathematics, 16–22 November 1969.

Döring, B.: Ein Satz über eine von Grebenjuk betrachtete Klasse von Iterationsverfahren.
Gekeler, E.: Relaxation bei einer Klasse nichtlinearer Gleichungssysteme.
Niethammer, W.: Konvergenzbeschleunigung bei einstufigen Iterationsverfahren durch Summierungsmethoden.
Wacker, H. J.: Eine Lösungsmethode zur Behandlung nichtlinearer Randwertprobleme.

A. I. van de Vooren.

G. Doetsch, **Einführung in Theorie und Anwendung der Laplace-Transformation,** 2nd edition, Birkhäuser Verlag, Basel, Stuttgart, 1970. 351 pp., price sFr. 78,–.

This is the second edition of the well-known textbook on Laplace transforms, especially written for applied mathematicians, engineers and physicists. Contrary to the three volumes of "Handbuch der Laplace-Transformation" by the same author, the treatment in the present work proceeds from the elements to the more complicated parts of the theory, and many instructive examples are given. The book is excellently suited for selfstudy.

Apart from a large number of minor alterations with respect to the first edition, there are two important additions in this second edition. First, there are new paragraphs dealing with the Laplace transform of generalized functions (distributions). The treatment is restricted to distributions of finite order, which makes a clear exposition possible for engineers and physicists. Extensive use of this new theory is made in the chapters on the solution of ordinary differential equations. Second, more attention is given to the complex inversion integral for Laplace transforms, especially when needed for the determination of the asymptotic behaviour of the original function. A subject which has been omitted in this second edition is the solution of difference equations.

H. W. Hoogstraten

J. K. Hale, **Ordinary Differential Equations,** Wiley-Interscience, New York, London, 1970. xvi+332 pages, price 140 sh.

This book presents a modern treatise on the theory of ordinary differential equations with emphasis on non-linear problems. Throughout the work many concepts from functional analysis are used freely and in the introductory Chapter 0 the most important tools, such as Banach spaces, linear transformations and fixed point theorems are recapitulated.

In Chapter I the fundamental theory of existence, uniqueness and continuous dependence is given. Chapter II deals with the global theory of two-dimensional systems. Chapters III and IV deal with linear and perturbed linear systems. Chapter V gives the method of averaging for non-linear oscillations with extensive applications on the Van der Pol and Duffing equations. Chapter VI deals with the behaviour near a periodic orbit and Chapter VII with integral manifolds of equations with a small parameter. Chapter VIII is devoted to a general method for determining when a periodic differential equation containing a small parameter has a periodic solution, and the abstract generalization of this method is made in Chapter IX with an application to analytic solutions of linear systems with a singularity. The last chapter is devoted to elementary results and applications of the direct method of Liapounov to stability theory. There is an Appendix on almost periodic functions. Nearly all chapters end with a section "Remarks and suggestions for further study" and there are many exercises.

The book is recommended to anyone who wishes to study the theory of non-linear ordinary differential equations on an advanced level.

H. W. Hoogstraten

A. Ghizzetti and A. Ossicini: **Quadrature Formulae,** International Series of Numerical Mathematics, Vol. 13. Birkhäuser Verlag, Basel, 1970. 192 pages, price sFr. 36,—.

The central theme of this book is to establish quadrature formulae of the following type

$$\int_a^b g(x)u(x)dx = \sum_{h=0}^{n-1} \sum_{i=1}^{m} A_{hi} u^{(h)}(x_i) + R[u(x)] .$$

The function $g(x)$ should be Lebesgue-integrable and the function $u(x)$ $n-1$ times continuously differentiable in $[a, b]$. The constant coefficients A_{hi} are independent of $u(x)$ and the linear functional $R[u(x)]$ (remainder) is zero when $u(x)$ is a solution of the homogeneous linear differential equation $E[u(x)]=0$.

When one wants a formula which is exact for a certain class of functions, this requirement determines the differential operator E. For instance, when the formula should be exact for polynomials of degree $n-1$, the operator E becomes d^n/dx^n. Then one chooses a set of arbitrary solutions of the differential equation $E^*(\phi)=0$, where E^* is the adjoint operator of E, and this set determines the coefficients A_{hi} as well as the functional R.

In order to obtain formulae which only contain u but not its derivatives in the points x_i, the solutions ϕ should satisfy the conditions $\phi(x_i)=0$.

One can derive by this method an immense variety of quadrature formulae, among them the classical ones such as the Newton–Cotes and the Gauss type formulae.

The interval may also be $[a, \infty)$ or $(-\infty, \infty)$.

The method is based upon the Green–Lagrange identity and upon relations between solutions of a linear differential equation and those of the adjoint equation. These properties are treated in the first chapter.

The general method is presented in Ch. 2, while Ch. 3 contains information on special functions. This is required for Ch. 4, where the method is applied to derive various quadrature formulae. Ch. 5 deals with generalized quadrature formulae which are the formulae obtained when the interval must be divided into a number of subintervals in order to increase the accuracy. To each of the subintervals the formulae of Ch. 2 are applied. This gives some complications. It is shown that if the formula is exact for $u(x)\equiv 1$, the division into ν subintervals leads to a convergent procedure if $\nu\to\infty$.

Each chapter contains a number of problems; their solutions are given in Ch. 6.

This is an interesting book, which may be of importance if a quadrature formula is needed that is exact for a certain class of functions.

A. I. van de Vooren